1. A line $x = 3$, $z = -1$ carries charge 20nC/m. A plane $x = -2$ carries charge 4nC/m^2 Draw a picture and then find the force on a point charge -5mC at the origin. [10] †

† Numbers in square brackets are the point values ascribed to the questions being asked.

2. In electromagnetics one often needs to find the distance between two points. Between two points defined by position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$, let the distance be d then the following follows.

$$d = |\mathbf{r}_2 - \mathbf{r}_1|$$

Coordinate points being defined as (x, y, z) in Cartesian-, (ρ, ϕ, z) in cylindrical- and (r, θ, ϕ) in spherical systems, show [5] that respectively for these systems,

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2,$$

$$d^2 = \rho_2^2 + \rho_1^2 - 2\rho_1\rho_2\cos(\phi_2 - \phi_1) + (z_2 - z_1)^2,$$

and

$$d^2 = r_2^2 + r_1^2 - 2r_1r_2\cos\theta_1\cos\theta_2 - 2r_1r_2\sin\theta_1\sin\theta_2\cos(\phi_2 - \phi_1).$$

3. An infinite plane in freespace has the charge density $\rho_S \mathrm{C} \cdot \mathrm{m}^{-2}$ and the equation

$$ax + by + cz = d$$

where a, b, c and d are constants. Find the unit vector $\mathbf{a}_n$ normal to this plane. [6] Then find the electric field intensity $\mathbf{E}$ and the electric flux density $\mathbf{D}$. [4]

4. In free space,p

$$\mathbf{D} = 2y^2\mathbf{a}_x + 4xy\mathbf{a}_y - \mathbf{a}_z \qquad \mathrm{mC/m^2}$$

Find the total charges stored in the region $1 < x < 2$, $1 < y < 2$ and $-1 < z < 4$. [5]

5. A vector field in mixed coordinate variables

$$\mathbf{G} = \frac{x \cos\phi}{\rho}\mathbf{a}_x + \frac{2yz}{\rho^2}\mathbf{a}_y + \left(1 - \frac{x^2}{\rho^2}\right)\mathbf{a}_z.$$

Express $\mathbf{G}$ in spherical system. [10]

6. Let $\int_A^B \mathbf{F} \cdot \mathbf{dl}$ be the work done in moving a particle from A to B, and the force field

$$\mathbf{F} = 2xy\mathbf{a}_x + (x^2 - z^2)\mathbf{a}_y - 3xz^2\mathbf{a}_z$$

acts on a particle travelling from $A(0,0,0)$ to $B(2,1,3)$. Find the work done for each case of the following paths.

a. $(0,0,0) \rightarrow (0,1,0) \rightarrow (2,1,0) \rightarrow (2,1,3)$ [5]

b. straight line from $(0,0,0)$ to $(2,1,3)$. [5]